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# Non-abelian Born-Infeld revisited

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**Abstract**

We discuss the non-abelian Born-Infeld action, including fermions, as a series in $\alpha'$. We review recent work establishing the complete result to ${\alpha'}^2$, and its impact on our earlier attempts to derive the Born-Infeld action using $\kappa$-symmetry.

## 1 Introduction

The Born-Infeld action for the Maxwell field was originally introduced to obtain a finite electrostatic energy for the electron. This is achieved by constructing a nonlinear theory for the Maxwell field with an upper bound on the allowed electric field. With just this requirement the theory is far from unique. In his review [1] Born indicates that he prefers the following action:

$$\mathcal{L}_{\mathrm{BI}} = -\sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})}\,. \tag{1}$$

His reasons for this choice are that the combination of metric and fieldstrength suggests a geometric interpretation, and that this particular example satisfies the requirement of electric-magnetic duality: the equations of motion are symmetric under the exchange of electric and magnetic fields.

These two properties are partly responsible for the fact that the Born-Infeld action reappeared in string theory, as the effective action for the open (super)string. The Born-Infeld action is also the effective theory for the vector field on the worldvolume of a D-brane. For $p < 9$ this requires a generalization of (1) with worldvolume scalar fields describing the transverse modes of the D-brane. For $p = 9$, the spacetime filling brane, the ordinary Born-Infeld theory is recovered. In [2] the complete result for $p = 9$ was presented.

In the case of $n$ parallel branes the gauge invariance is extended to $U(1)^n$. Open strings stretching between the branes give rise to massive vector fields, which however become massless when all branes overlap. In that case there are $n^2$ massless vector fields on the common worldvolume, and the gauge invariance is enhanced to $U(n)$ [3]. The worldvolume theory for a system of $n$ overlapping D9-branes is called the non-abelian Born-Infeld theory,

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and coincides with the effective theory for open superstrings with $U(n)$ Chan-Paton factors. For $p < 9$ transverse scalars, also in the adjoint representation of $U(n)$, appear.

The construction of the non-abelian Born-Infeld theory has turned out to be a difficult problem [4]. However, a lot of progress has been made this year and in the following sections we will give an overview of recent work. In Sect. 2 we will expand the Born-Infeld theory in $\alpha'$, and discuss symmetric traces. In Sect. 3 we discuss the recent results of [5] on $\alpha'^2$, and our string theory calculation [6] at this order. We compare with a $\kappa$-symmetry [7] approach in Sect. 4, and discuss higher orders in $\alpha'$ in Sect. 5.

## 2 The $\alpha'$ expansion and symmetric traces

In this paper we will discuss the Born-Infeld theory as a nonlinear deformation of the supersymmetric Yang-Mills theory in $d = 10$. We define fields such that the dimension of the fieldstrength $\dim F = 2$, and the adjoint Majorana-Weyl fermion $\chi$ has $\dim \chi = 3/2$. The action then takes on the form

$$\mathcal{L} = \frac{1}{g^2}\left(F^2 + \alpha' F^3 + \alpha'^2 F^4 + \alpha'^3 F^5 + \ldots\right), \tag{2}$$

where in each order in $\alpha'$ we should add the fermionic partners, keeping in mind that $\dim \alpha'^2 \bar\chi \partial \chi = 0$. The Yang-Mills coupling constant $g$ plays no role in the following and can be set equal to one.

The terms at $\alpha'^0$ are

$$\mathcal{L}_0 = -\tfrac{1}{4} F^{\mu\nu\,a} F_{\mu\nu}{}^a + \tfrac{1}{2}\bar\chi^a \not{D} \chi^a , \tag{3}$$

where the indices $a, b, \ldots$ indicate the $U(n)$ adjoint representation. The action (3) is invariant under the following supersymmetry transformations:

$$\delta_0 A_\mu{}^a = \bar\epsilon \gamma_\mu \chi^a , \qquad \delta_0 \chi^a = \tfrac{1}{2}\gamma^{\mu\nu}\epsilon F_{\mu\nu}{}^a . \tag{4}$$

The question is then how to extend (3) to higher orders in $\alpha'$ in a supersymmetric way. It is known from string theory that there is no need for an order $\alpha'$ contribution. From the supersymmetry point of view one finds that all possible terms at order $\alpha'$ can be redefined away or violate supersymmetry. A first result on the $\alpha'^2$ contribution was given in [8]. There it was shown that a supersymmetric extension can be obtained if the terms at order $\alpha'^2$ are written as a symmetric trace, that is, the terms $F^4$ should be written as

$$S_{abcd}\, F^a F^b F^c F^d \equiv \operatorname{Str} F^4 , \quad S_{abcd} \equiv \operatorname{tr} T_{(a} T_b T_c T_{d)} . \tag{5}$$

(where the parentheses indicate symmetrization) and similarly for the terms bilinear in the fermions (quartic fermions were not considered in [8]). With this symmetric trace the $\alpha'^2$ terms are a straightforward generalization of the abelian result [8].

The discussion about non-abelian Born-Infeld has been dominated by the possibility that the complete result might be symmetric trace (see [9] for a careful discussion of this issue). The full result at order $\alpha'^n$ must be of the form

$$S = S_1 + S_2 + S_3 , \tag{6}$$

with

$$S_1 \sim \operatorname{Str} F^{n+2}\,, \quad S_2 \sim F^n[F,F]\,, \quad S_3 \sim F^n\{D,D\}F\,, \tag{7}$$

that is, all terms which are not written as a symmetric trace must contain commutators of fields (or of derivatives of fields). In case of higher derivative terms, as in $S_3$, commutators of derivatives need not appear since they can be rewritten as terms in $S_2$. Of course more derivatives than indicated in $S_3$ may appear. It was known that for the terms given in [8] at order $\alpha'^2$ $S_2 = S_3 = 0$. But is was also known from earlier results from string theory [10] that $S_3$ terms are needed at order $\alpha'^3$ and that they are not in the form of a symmetric trace.

Therefore the conjecture that the full answer is a symmetric trace cannot be true, a statement that will derive further support from results to be discussed below. However, it is still possible that the symmetric trace part of $S$ will turn out to be a useful approximation to the full result.

## 3 The order $\alpha'^2$ terms

The full order $\alpha'^2$ action including quartic fermions was recently constructed with superspace methods [5]. The result is, in our notation,

$$\begin{aligned}\mathcal{L}_2 \;=\; & S^{abcd}\big[\tfrac{1}{8}F_{\mu\nu}{}^a F_{\lambda\rho}{}^b F^{\mu\lambda\,c}F^{\nu\rho\,d} - \tfrac{1}{32}F_{\mu\nu}{}^a F^{\mu\nu\,b}F_{\lambda\rho}{}^c F^{\lambda\rho\,d} \\ & -\tfrac{1}{4}F_{\mu\nu}{}^a F^{\mu\rho\,b}\bar\chi^c\gamma^\nu D_\rho\chi^d - \tfrac{1}{16}F^{\mu\nu\,a}D_\mu F^{\lambda\rho\,b}\bar\chi^c\gamma_{\nu\lambda\rho}\chi^d \\ & +\tfrac{1}{24}\bar\chi^a\gamma^\mu D^\nu\chi^b\bar\chi^c\gamma_\mu D_\nu\chi^d + f^{def}\big(-\tfrac{7}{192}F^{\mu\nu\,a}\bar\chi^b\gamma_{\mu\nu\rho}\chi^c\bar\chi^e\gamma^\rho\chi^f \\ & +\tfrac{1}{1152}F^{\mu\nu\,a}\bar\chi^b\gamma^{\lambda\rho\sigma}\chi^c\bar\chi^e\gamma_{\mu\nu\lambda\rho\sigma}\chi^f\big)\big]\,.\end{aligned} \tag{8}$$

One sees that the result is a symmetric trace, except for two four-fermion terms. In a recent paper [6] we have calculated the open string four-point function. This allows the calculation of terms quartic in the basic fields in the effective action. This calculation also gives the order $\alpha'^2$ action except the last terms in (8) which are a product of five fields. Let us briefly recall the basics of such a string calculation. The four-point function is given by the following formula:

$$A_4 = -8ig^2\,K(1,2,3,4)\,\big(T_{1\,abcd}G(s,u) + T_{2\,abcd}G(s,t) + T_{3\,abcd}G(t,u)\big)\,, \tag{9}$$

where $g$ is the Yang-Mills coupling constant and $s,t$ and $u$ are the standard Mandelstam variables satisfying $s+t+u=0$. $K$ contains the wave-functions of the external fermion lines, the $T_i$ contain the traces over $U(n)$ generators, and satisfy in particular the relation

$$(T_1+T_2+T_3)_{abcd} = S_{abcd}\,. \tag{10}$$

$G$ is the Veneziano amplitude

$$G(s,t) \;=\; \frac{1}{st}\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha'(s+t))} = \frac{1}{st} - \frac{\pi^2\alpha'^2}{6} - \zeta(3)u\alpha'^3 + \dots \tag{11}$$

From this it is already clear that the four-point function at order $\alpha'^2$ contains a symmetric trace over the Yang-Mills indices. For higher orders in $\alpha'$ this will not occur. In the absence of order $\alpha'$ terms this implies that the quartic effective action at order $\alpha'^2$ is also a symmetric trace, and our result agrees completely with the result of [5].

Clearly the four-point function also has contributions at higher orders in $\alpha'$. Certain higher-derivative terms in the effective action at order $\alpha'^3$ were recently constructed from this part of the four-point function [11].

## 4 The fate of $\kappa$-symmetry

In a recent paper [7] we proposed a method to obtain the non-abelian Born-Infeld action using $\kappa$-symmetry. In such a construction one starts with an $N = 2$ supersymmetric theory on the worldvolume. The fields are (in the abelian case) the embedding coordinates $X^\mu(\sigma)$ of the brane, a 32-components target space spinor $\theta(\sigma)$, and the worldvolume vector $A_i(\sigma)$. The symmetries acting on these fields are a global $N = 2$ supersymmetry (parameter $\epsilon$), a local $N = 2$ supersymmetry with parameter $\kappa(\sigma)$, abelian gauge transformations and worldvolume reparametrisations with parameters $\xi^i(\sigma)$. On the fermions $\theta$ the fermionic symmetries act as

$$\delta\theta = \alpha'^{-1}\big( - \epsilon + (1 + \Gamma)\kappa\big)\,, \tag{12}$$

where $\Gamma$ is a field-dependent object satisfying $\Gamma^2 = 1$. This implies that by gauge fixing $\kappa$-symmetry half of the fermions can be gauged away, leaving only the $N = 1$ fermion $\chi$. The worldvolume reparametrisations can be gauge fixed by identifying the embedding coordinates with the worldvolume coordinates $\sigma$ (the static gauge). After this gauge fixing two global supersymmetries remain, one realized linearly (the parameter $\epsilon$ from previous sections), the other realized nonlinearly.

In the abelian case, $\kappa$-symmetry has been very useful in obtaining the effective action for single Dp-branes in flat [2] and curved backgrounds [12]. In particular, the complete result for the abelian $D9$-brane was given in [2].

In our approach for the non-abelian case we start with a non-abelian $\kappa$-symmetry. The idea is that we now have $n^2$ type II fermions on the worldvolume, and to the reduce all of these to 16-component type I fermions we need $n^2$ local fermionic symmetries, i.e., $\theta$ should transform as

$$\delta\theta^a = \alpha'^{-1}\big( - \epsilon^a + (\delta^{ab} + \Gamma^{ab})\kappa^b\big)\,, \tag{13}$$

again with $\Gamma^2 = 1$. The parameter $\epsilon^a$ is constant, and therefore has to satisfy $f^{abc}\epsilon^c = 0$. Early on in our analysis we discovered that our approach could only work in the static gauge, so there are no embedding coordinates $X^\mu$.

We obtained a $\kappa$-symmetric action of the following schematic form:

$$\mathcal{L} = F^2 + \bar\theta\partial\theta(1 + \alpha' F + \alpha'^2 F^2) \tag{14}$$

Invariance under $\kappa$-symmetry requires

$$\delta A = \bar\kappa(1 + \alpha' F + \ldots)\gamma\theta \tag{15}$$

We checked the $\kappa$-invariance to order $\alpha'$ in the variation. This gave us only the terms at order $\alpha'^2$ with fermions (14). It turned out that these were not in the form of a symmetric trace, which is in contradiction with the results of the previous section. There are now three possibilities:

- Note that (14) also contains terms of order $\alpha'$. These can be redefined away, and it might be that this redefinition modifies the $\alpha'^2$ terms to a symmetric trace.
- It might be that supersymmetry does not fix the action uniquely, and that our $\kappa$-symmetric action leads a different supersymmetric invariant (which then would have no obvious relation to string theory).
- $\kappa$-symmetry, based on our assumptions, cannot be extended to order $\alpha'^2$.

We have checked that the first two possibilities are not realized, so we conclude that at least one of the assumptions used in deriving (14) is false. Let us briefly review the main assumptions:

- $\kappa$-symmetry is non-abelian, each type I fermion is doubled. This still seems the most reasonable starting point to us. A proposal with only one $\kappa$-symmetry in a non-abelian Born-Infeld action has been proposed in [13].
- For $n$ overlapping branes we use only one set of worldvolume coordinates and embedding coordinates (before going to the static gauge). In principle one could think of a more ambitious approach, which starts with as many coordinate systems and embedding coordinates as there are branes. The difficulty here is to impose the enlarged (non-abelian?) reparametrization invariance.

Presently it is not clear how to proceed with $\kappa$-symmetry in the non-abelian context. Because the nonlinear supersymmetry is still present at order $\alpha'^2$ it is neverthless tempting to believe that the results of the previous section are the gauged fixed version of formulation with a larger, local symmetry.

## 5 Higher orders in $\alpha'$

For a long time the only information on orders $\alpha'^k,\ k > 2$ came from [10]. Recently more details about the case $\alpha'^3$ have been obtained, while some partial results for order $\alpha'^4$ are also known.

Let us start with $\alpha'^3$. In [14] the one-loop bosonic five-point function in supersymmetric $N = 4\ d = 4$ Yang-Mills theory is evaluated. The corresponding contribution to the effective action must be supersymmetric. If supersymmetry uniquely determines this term in the effective action, then it should be the same as the corresponding contribution to the non-abelian Born-Infeld theory.

In [15] the bosonic terms at $\alpha'^3$ have been determined by requiring that a particular BPS solution, related to branes intersecting at angles [16], of the $\alpha'^0$ field equations, remains a solution of the deformation of the Yang-Mills theory with nonlinear terms.

There are two types of terms in the result of [14, 15], those with $F^5$ and the terms of type $F^2(DF)^2$. In ten dimensions there six independent $F^5$ terms, while there are only four in $d = 4$. The $d = 4$ result therefore leads to a two-parameter solution in $d = 10$. For the terms with derivatives there is no such ambiguity. Indeed, all authors [10, 11, 14, 15] agree on the the $F^2(DF)^2$ terms. For the $F^5$ terms the free parameters in the ten-dimensional version of [14] cannot be chosen in such a way that agreement with [15] is obtained. The ordering of the Yang-Mills fields in the [10] result is not sufficiently specified to make a comparison with the new results possible.

Although we still seem to be far from a complete result, a lot of information has become available in the past year, and more is likely to appear next year. Hopefully the new results will reveal sufficient structure to recognize the form of the complete answer.

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## References

[1] M. Born, *Théorie non-linéaire du champ électromagnétique*, Ann. de l'Institut Henri Poincaré **7** (1937) 155

[2] M. Aganagic, C. Popescu and J.H. Schwarz, *Gauge-Invariant and Gauge-Fixed D-Brane Actions*, Nucl. Phys. **B495** (1997) 99, [hep-th/9612080].

[3] E. Witten, *Bound States of Strings and $p$-Branes*, Nucl. Phys. **B460** (1996) 35, [hep-th/9510135].

[4] A review of Born-infeld theory can be found in A.A. Tseytlin, *Born-Infeld Action, Supersymmetry and String Theory*, to appear in the Yuri Golfand memorial volume, [hep-th/9908105].

[5] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *The structure of maximally supersymmetric Yang-Mills theory: constraining higher-order corrections*, JHEP **0106** (2001) 034, [hep-th/0102009]; *D=10 super Yang-Mills at $\mathcal{O}(\alpha'^2)$*, JHEP **0107** (2001) 042, [hep-th/0104236].

[6] E.A. Bergshoeff, A. Bilal, M. de Roo and A. Sevrin, *Supersymmetric non-abelian Born-Infeld revisited*, JHEP **0107** (2001) 029, [hep-th/0105274].

[7] E.A. Bergshoeff, M. de Roo and A. Sevrin, *Non-abelian Born-Infeld and kappa-symmetry*, J. Math. Phys. **42** (2001) 2872, [hep-th/0011018].

[8] E. Bergshoeff, M. Rakowski and E. Sezgin, *Higher-derivative super Yang-Mills theories*, Phys. Lett. **B185** (1987) 371

[9] A.A. Tseytlin, *On Non-Abelian Generalization of Born-Infeld Action in String Theory*, Nucl. Phys. **B501** (1997) 41, [hep-th/9701125].

[10] Y. Kitazawa, *Effective lagrangian for open superstring from five point function*, Nucl. Phys. **B289** (1987) 599

[11] A. Bilal, *Higher-derivative corrections to the non-abelian Born-Infeld action*, Nucl. Phys. **B618** (2001) 21, [hep-th/0106062].

[12] M. Cederwall, A. von Gussich, B. E. W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet Super-p-Branes in Type IIA and IIB Supergravity*, Nucl. Phys. **B490** (1997) 179, [hep-th/96100148]; E. Bergshoeff and P. K. Townsend, *Super-D-Branes*, Nucl. Phys. **B490** (1997) 145, [hep-th/9611173].

[13] D. Sorokin, *Coincident (Super-)Dp-Branes of Codimension One*, [hep-th/0106212].

[14] A. Refolli, A. Santambrogio, N. Terzi and D. Zanon, *$F^5$ contribution to the non-abelian Born Infeld action from a supersymmetric Yang-Mills five-point function*, Nucl. Phys. **B613** (2001) 64, [hep-th/0105277].

[15] P. Koerber and A. Sevrin, *The non-abelian open superstring effective action through order $\alpha'^3$*, JHEP 0110 (2001) 003, [hep-th/0108169].

[16] M. Berkooz, M. R. Douglas and R. Leigh, *Branes intersecting at angles*, Nucl. Phys. **B480** (1996) 265, [hep-th/9606139].